Math Diversion Problem 377

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> Beauty is the first test: there is no permanent place
> in the world for ugly mathematics.
> — G.H. Hardy

1 The Problem

This problem comes from the website

`https://www.algebra.com/algebra/homework/Rate-of-work-word-problems/Rate-of-work-word-problems.faq.question.980634.html#google_vignette`

(Problem statement)

> `Question 980634: Together, Diane and Craig can mow their lawn in 1 hour and 10 minutes. Working alone, Diane can mow the lawn in 2 hours. How long will it take Craig to mow the lawn when working alone?`

2 The Preparation

See my write-ups on mixed-rate problems from my prior word-problem solutions.

3 The Solution

Question: Are there any totals presented in this problem? Yes. There is a total of one job being done by two people (Diane and Craig) and the same job being done by one person (Diane). Okay, if we let the rate at which Diane and Craig work on the job to be R_D and R_C, respectively, then the two equations representing the 'total equaling the sum of its parts' are given by

$$1 \text{ job} = R_D T_D + R_C T_C = (R_D + R_C)T\,, \tag{1a}$$

$$1 \text{ job} = R_D T'_D\,, \tag{1b}$$

where the first equation represents the scenario when both work together for a time T, and the second represents when Diane works alone for a time 2 hrs = 120 minutes = T'_D. It's simple to solve for R_D from Eq. (1b):

$$R_D = \frac{1\,\text{job}}{T'_D} = \frac{1\,\text{job}}{120\,\text{min}}\,. \tag{2}$$

And now we can solve (1a) for R_C (with $T = 70\,\text{min}$).

$$R_C = \frac{1}{T} - R_D = \frac{1}{70} - \frac{1}{120} = \frac{1}{168}\,\left[\frac{\text{job}}{\text{min}}\right]\,. \tag{3}$$

So, for Craig to mow the lawn by himself, he would need

$$168\,\text{min} = 2.8\,\text{hr} = 2\,\text{hr}\ 48\,\text{min}\ . \tag{4}$$