Math Diversion Problem 379

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> The ignorant always loud in argument.
> — Charlie Chan

The YouTube video is found at:

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Source: https://www.youtube.com/shorts/oxdDcIjdGwE
Title: Can you solve this 'quite tough' equation?
 Presenter: Gretsy Academy
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1 The Problem

Given the relation

$$W(x) = \ln(5x)\,, \tag{1}$$

find the values of x.

[Skip down to the solution, if you prefer.]

2 The Preparation

A lemma I'll need from the theory of the Lambert W function is the following: If

$$y \ln y = B\,, \tag{2}$$

then

$$\ln y = W(y \ln y) = W(B)\,. \tag{3}$$

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3 The Solution

I know of two ways to extricate oneself from inside a Lambert W function. The first is by its definition:

$$W(ye^y) = y\,, \tag{4}$$

and the second is by this alternate

$$W(y \ln y) = \ln y \,. \tag{5}$$

Let's use this second way. Let

$$x = y \ln y \,. \tag{6}$$

Then (1) becomes

$$\ln y = \ln (5y \ln y) \,. \tag{7}$$

After dropping the logarithms, we get

$$y = 5y \ln y \,. \tag{8}$$

Canceling the y's, we can solve for y:

$$y = e^{1/5} \,. \tag{9}$$

And finally,

$$x = \tfrac{1}{5} e^{1/5} \,. \tag{10}$$