Math Diversion Problem 380

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All my life I kept running into smart people.... In school
there were lots of smarter kids. And when I first joined the
force, sir, they had some very clever people there. And I
could tell right away that it wasn't going to be easy making
detective as long as they were around. What I figured that...
if I worked harder than they did. Put in more time. Read
the books. Kept my eyes open. Maybe I could
make it happen. And I did!
— Lt. Columbo to his prisoner
(from the TV show *Columbo*)
("The Bye Bye Sky High)
IQ Murder Case")

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=NqCShAtKawY
Title: A Curious Exponential Equation
      | Problem 227
 Presenter: aplusbi
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1 The Problem

Given the relation

$$8^z = -1 + \sqrt{3}i\,, \tag{1}$$

find the values of z.

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2 The Solution

Let's begin by reframing the RHS of (1):

$$8^z = 2\big(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\big) = 2e^{2\pi i/3}\,. \tag{2}$$

Now we take the logarithm across this equation:

$$z \ln 8 = \ln 2 + 2\pi i/3 + 2\pi i n \quad \text{where} \quad n \in \mathbb{Z} \,. \tag{3}$$

Dividing through by $\ln 8$, we get

$$z = \frac{\ln 2}{\ln 8} + \frac{2\pi i/3 + 2\pi i n}{\ln 8} \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{4}$$

or

$$z = \frac{1}{3} + \frac{2\pi i/3 + 2\pi i n}{\ln 8} \quad \text{where} \quad n \in \mathbb{Z} \,, \tag{5}$$

where we used that $\ln 8 = 3 \ln 2$.