Math Diversion Problem 382

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> In my attempting to solve these 'olympiad'-style problems,
> I have noticed that a great many of the more difficult-
> looking problems are really just cleverly concealed
> quadratic or cubic polynomial equations.
> — The Author

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=wCbkVVkG_vE
Title: Stanford University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relation

$$16^x + 44^x = 121^x \,, \tag{1}$$

find the (real) values for x.

2 The Solution

The best way to go here is to divide through by 16^x, then,

$$1 + \left(\frac{44}{16}\right)^x = \left(\frac{121}{16}\right)^x \,, \tag{2}$$

or

$$1 + \left(\frac{11}{4}\right)^x = \left(\frac{11^2}{4^2}\right)^x \,. \tag{3}$$

Let

$$\beta = \left(\frac{11}{4}\right)^x \,. \tag{4}$$

Then (2) becomes the quadratic in β:

$$1 + \beta = \beta^2 \,, \tag{5}$$

which has solutions

$$\beta = \frac{1 \pm \sqrt{5}}{2} \,. \tag{6}$$

Hence, the real solution for x is

$$x = \frac{\log\left[\frac{1}{2}(1+\sqrt{5})\right]}{\log\left(11/4\right)} \,, \tag{7}$$

where I threw out the negative value in (6).