Math Diversion Problem 395

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February 8, 2025

> Don't ever take a fence down until you know
> the reason it was put up.
> — Chesterton

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=DC9Xo_GV_ug
Title: Germany | Solving Log With Different Bases
 Presenter: Math Hunter
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1 The Problem

Given the relation

$$\log_2 x + \log_3 x = 5\,, \tag{1}$$

find the values of x.

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2 The Solution

Let's put all the logarithms into the same base, base 10 will do. Then (1) becomes

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 3} = 5\,, \tag{2}$$

which can be factored,

$$\log x \left[\frac{1}{\log 2} + \frac{1}{\log 3}\right] = 5\,. \tag{3}$$

Then

$$\log x \left[\frac{\log 2 + \log 3}{(\log 2)(\log 3)}\right] = 5\,. \tag{4}$$

Then,

$$\log x = 5 \left[\frac{(\log 2)(\log 3)}{(\log 6)} \right] . \tag{5}$$

Finally,

$$x = 10^{5\left[\frac{(\log 2)(\log 3)}{(\log 6)} \right]} . \tag{6}$$

However, WolframAlpha is not happy with this answer. It makes a few subtle changes. First

$$x = 10^{\left\lceil \frac{(\log 2)(\log 3^5)}{(\log 6)} \right\rceil} = 10^{\left\lfloor \frac{(\log 2)(\log 243)}{(\log 6)} \right\rfloor} . \tag{7}$$

Then

$$x = \left(10^{(\log 2)}\right)^{(\log 243)/(\log 6)} . \tag{8}$$

And finally,

$$x = 2^{(\log 243)/(\log 6)} . \tag{9}$$