

Math Diversion Problem 400

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There still remains one point to be cleared up. One of the most fundamental questions has not been settled as yet: does an inertial system exist? We have learned something about the laws of nature, their invariance with respect to Lorentz transformation, and their validity for all inertial systems moving uniformly relative to each other. We have the laws but we do not know the frame to which to refer them.
— Albert Einstein

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=FFC2Ts1GamQ>
Title: Solving A Cubic System | Problem 335
Presenter: aplusbi

1 The Problem

Given the relations

$$z^2 + w^2 = 0, \tag{1a}$$

$$z^3 + w^3 = -4, \tag{1b}$$

find the values of z, w .

2 The Solution

So, I chose as my ‘first fundamental quantity’ and follow-up:

$$a = z + w, \tag{2a}$$

$$a^2 = (z + w)^2 = z^2 + 2zw + w^2 = 2zw, \tag{2b}$$

$$a^3 = (z + w)^3 = z^3 + 3z^2w + 3zw^2 + w^3 \tag{2c}$$

$$= 3zw(z + w) - 4 \tag{2d}$$

$$= \frac{3}{2}a^3 - 4, \tag{2e}$$

where we used (1a) in (2b), and used (1b) in (2d). Hence,

$$a^3 = 8. \tag{3}$$

If I had the patience for complications, I’d use all three complex roots to this last equation. Instead, I’ll settle for

$$a = 2. \tag{4}$$

Therefore,

$$(z + w)^2 = 4, \tag{5}$$

from which we get that

$$zw = 2. \tag{6}$$

Using this last equation and that $z + w = 2$, we get for z, w :

$$z = 1 \pm i, w = 1 \mp i. \tag{7}$$