

Math Diversion Problem 402

P. Reany

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After a time, you may find that having is not so
pleasing a thing after all as wanting. It is not
logical, but is often true.
— Spock

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=TavEDj8N00g>
Title: An Exponent That Triples | Problem 380
Presenter: aplusbi

1 The Problem

Given the relation

$$i^z = 3i, \quad (1)$$

find the values for z .

2 The Solution

Let's begin by taking the natural logarithm across (1):

$$z \ln i = \ln 3i + 2ni\pi \quad \text{where } n \in \mathbb{Z}. \quad (2)$$

But $\ln i = \ln e^{i\pi/2} = i\pi/2$.

$$z \ln i = \ln 3i + 2ni\pi \quad \text{where } n \in \mathbb{Z} \quad (3)$$

$$= \ln 3 + \ln i + 2ni\pi \quad \text{where } n \in \mathbb{Z} \quad (4)$$

Then,

$$z = \frac{\ln 3 + \ln i + 2ni\pi}{\ln i} \quad \text{where } n \in \mathbb{Z} \quad (5)$$

$$= \frac{\ln 3 + 2ni\pi}{i\pi/2} + 1 \quad \text{where } n \in \mathbb{Z} \quad (6)$$

$$= -\frac{2i \ln 3}{\pi} + 4n + 1 \quad \text{where } n \in \mathbb{Z}. \quad (7)$$