Math Diversion Problem 408

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> Every big idea needs someone to defend it.
> — Cybersecurity

The YouTube video is found at:

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Source: ---
Title: ---
 Presenter: Math-x
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1 The Problem

Given the relation

$$2^b + b = 5\,, \tag{1}$$

find the values of b.

2 The Preparation

I intend to use the Lambert W function, which goes as follows: If

$$ze^z = B\,, \tag{2}$$

then

$$z = W(B)\,, \tag{3}$$

where there are domain constraints on B that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

I also intend to use the Lambert W function Lemma, that, for $a > 0$, given

$$za^z = B\,, \tag{4}$$

then

$$z = W_a(B)\,, \tag{5}$$

where

$$W_a(B) \equiv \frac{W(B \ln a)}{\ln a}\,, \tag{6}$$

which becomes the ordinary Lambert W function when $a = e$.

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3 The Solution

First, we rewrite the Given relation to

$$2^b = 5 - b\,, \tag{7}$$

and then set[1]

$$a \equiv 5 - b\,, \tag{8}$$

so that (7) becomes

$$2^{5-a} = 2^5 2^{-a} = a\,. \tag{9}$$

This last equation can be put into the canonical form:[2]

$$a2^a = 2^5\,. \tag{10}$$

Now we can use the lemma above.

$$a = W_2(2^5) = \frac{W(2^5 \cdot \ln 2)}{\ln 2}\,. \tag{11}$$

And finally,

$$b = 5 - a = 5 - \frac{W(32 \cdot \ln 2)}{\ln 2}\,. \tag{12}$$

[1] The a used here is not the same as used in the lemmas.

[2] By 'canonical form' I mean a form ready to use one of the lemmas.