Math Diversion Problem 410

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He that answereth a matter before he heareth it,
it is folly and shame unto him.
— Proverbs 18:13

Abstract

Here we treat a problem in specific heat as a mere algebra word problem.

1 Problem Statement

A 100 g gold-copper alloy sample has its temperature raised 23.4 °C by adding to it 200 calories of heat. The owner of the sample was told that the amount of copper in the sample is less than 50% by weight. Is this claim true or false?

2 Setup

Let Q_x be the heat added to a substance of mass m_x and specific heat C_x. The *specific heat* of a substance is the amount of heat energy needed to raise the temperature of one gram of the substance 1°C. The equation that relates these thermodynamic variables for element x is

$$Q_x = m_x C_x \Delta T \,, \tag{1}$$

where T is the temperature in kelvin or °C. Either scale will work because ΔT deals only in temperature differences. So, for gold (Au) and copper (Cu), we have that

$$Q_{\text{Au}} = m_{\text{Au}} C_{\text{Au}} \Delta T \quad \text{and} \quad Q_{\text{Cu}} = m_{\text{Cu}} C_{\text{Cu}} \Delta T \,, \tag{2}$$

where

$$C_{\text{Au}} = 0.0301 \,\text{cal/g} \cdot \text{K} \quad \text{and} \quad C_{\text{Cu}} = 0.0923 \,\text{cal/g} \cdot \text{K} \,. \tag{3}$$

3 Comments

If you think about it, assaying the ratio of one metal to another in an alloy could be a tedious and messy affair, especially if one needs to take a sample of the alloy and perform some sort of chemical tests on it. But this analysis only needs to know the mass of the alloy (which can be easily determined by weighing) and what two metals are in it, which we will take at face value. (Big assumption.) We start the experiment at an initial temperature and take it to the final temperature and record the heat consumed. The difference in temperature is ΔT.

As for the source of the problem, I either made it up or I misplaced the actual source. My apology to the actual source, if the latter is true. It's my practice to acknowledge sources when I know them.

4 Scheme & Solution

Are there any totals in the problem? (Almost always there are.)

First, there is a total of 100 g of alloy, which is in gold and copper. By additivity of mass, the total mass is the sum of its parts:

$$100 = m_{\mathrm{Au}} + m_{\mathrm{Cu}}\,. \tag{4}$$

And, of course, there's the total heat applied to the sample, and it is also additive in its distribution between the gold and copper (in thermal equilibrium).

$$\begin{aligned}
200 &= Q_{\mathrm{Au}} + Q_{\mathrm{Cu}} \\
&= m_{\mathrm{Au}} C_{\mathrm{Au}} \Delta T + m_{\mathrm{Cu}} C_{\mathrm{Cu}} \Delta T \\
&= \big(m_{\mathrm{Au}} C_{\mathrm{Au}} + m_{\mathrm{Cu}} C_{\mathrm{Cu}}\big)\Delta T \\
&= \big[m_{\mathrm{Au}} C_{\mathrm{Au}} + (100 - m_{\mathrm{Au}}) C_{\mathrm{Cu}}\big]\Delta T \\
&= \big[m_{\mathrm{Au}}(0.0301) + (100 - m_{\mathrm{Au}})(0.0923)\big](23.4)\,.
\end{aligned} \tag{5}$$

For the next step, we can write

$$\frac{200}{23.4} - (100)(0.0923) = m_{\mathrm{Au}}[(0.0301) - (0.0923)\,]\,. \tag{6}$$

For the last step, we get

$$m_{\mathrm{Au}} = \frac{8.547 - 9.23}{0.0301 - 0.0923} = 10.98\,[\mathrm{g}]\,, \tag{7}$$

So, this sample is about 11% gold and 89% copper. Therefore, the claim is false.