

# Math Diversion Problem 417

P. Reany

February 18, 2025

The greatest killer of creativity is interruption.  
— John Cleese

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=grE-NkoMP4M>  
time register 4:29  
Title: Top 7 math Olympiad Question  
Presenter: MindSphere

## 1 The Problem

Given the relation

$$2^x + x = 11, \tag{1}$$

find the real values of  $x$ .

## 2 The Preparation

I intend to use the Lambert  $W$  function, which goes as follows: If

$$ze^z = B, \tag{2}$$

then

$$z = W(B), \tag{3}$$

where there are domain constraints on  $B$  that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

I intend to use the Lambert  $W$  function Lemma, that, for  $a > 0$ , given

$$za^z = B, \tag{4}$$

then

$$z = W_a(B), \tag{5}$$

where

$$W_a(B) \equiv \frac{W(B \ln a)}{\ln a}, \quad (6)$$

which becomes the ordinary Lambert  $W$  function when  $a = e$ .

---

A lemma I'll need from the theory of the Lambert  $W$  function is the following:

If

$$y \ln y = B, \quad (7)$$

then

$$\ln y = W(y \ln y) = W(B). \quad (8)$$

### 3 The Solution

First, let's rewrite (1) to the following form:

$$2^x = 11 - x, \quad (9)$$

and then set

$$y = 11 - x, \quad (10)$$

hence, (9) becomes

$$2^{11-y} = y, \quad (11)$$

or

$$2^{11} = y2^y. \quad (12)$$

On employing the lemmas, we get

$$y = W_2(2^{11}) = \frac{W(2^{11} \ln 2)}{\ln 2} = \frac{W(2^8 \ln 2^3)}{\ln 2} = \frac{W(2^8 \ln 2^8)}{\ln 2} = \frac{\ln 2^8}{\ln 2} = 8. \quad (13)$$

Hence,

$$x = 3. \quad (14)$$