Math Diversion Problem 421

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> Equations are just the boring part of mathematics. I attempt to see things in terms of geometry.
> — Stephen Hawking

1 The Problem

Prove the relation: Let

$$f(x) = a^x \,, \tag{1}$$

where a is a constant, then show that

$$D_x a^x = a^x \ln a \,, \tag{2}$$

where D_x is the derivative with respect to x.

2 The Solution

The way I see this proof, there are two small tricks to it. The first one is to take the logarithm of (1):

$$\ln f = x \ln a \,. \tag{3}$$

Then we differentiate by x:

$$\frac{f'}{f} = \ln a \,. \tag{4}$$

The second trick is to remember that we're solving for f'.

$$f' = f \ln a \,, \tag{5}$$

or

$$D_x \, a^x = a^x \ln a \,. \tag{6}$$