Math Diversion Problem 426

P. Reany

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With me, everything turns into mathematics.
— Rene Descartes
(P.S. I calculate; therefore I am.)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=huPSxaz0H2I
Title: Nice Exponential Problem
 Presenter: Math Beast
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1 The Problem

Given the relation

$$x^{625} = 5^x \,, \tag{1}$$

find the values of x.

2 The Solution

Note: $625 = 5^4$.

Let's try an α substitution to (1). Let

$$x = 5^\alpha \,, \tag{2}$$

then (1) becomes

$$(5^\alpha)^{625} = 5^{5^\alpha} \,, \tag{3}$$

or

$$\alpha 5^4 = 5^\alpha \,, \tag{4}$$

where it looks like $\alpha = 5$. Therefore,

$$x = 5^5 = 3125 \,. \tag{5}$$