Math Diversion Problem 430

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Theory like mist on eyeglasses — obscures facts.
— Charlie Chan

The YouTube video is found at:

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Source: https://indico.cern.ch/event/726779/contributions
/2991244/attachments/1642552/2727515
/complex_numbers_exercises.pdf
Title: Complex numbers- Exercises with detailed solutions
 Presenter:  CERN
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1 The Problem

Compute the square root of

$$z = -1 - i\,. \tag{1}$$

2 The Solution

I'll take advantage of the fact that

$$-1 - i = -1 \cdot (1 + i) = e^{i\pi} \cdot \sqrt{2}\,\frac{1+i}{\sqrt{2}} = \sqrt{2}\,e^{i\pi}e^{i\pi/4} = \sqrt{2}\,e^{5i\pi/4}\,. \tag{2}$$

Before I complete my taking of the square root, I'll present what Wolfram Alpha has to say about it: When I entered

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'z=-1-i, solve for z^{1/2}'
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it returned only one root:

$$\sqrt[4]{2}\,e^{-(3i\pi)/8}\,. \tag{3}$$

Anyway, my solutions are

$$z^{1/2} = \begin{cases} +\sqrt[4]{2}\,e^{5i\pi/8}\,, \\ -\sqrt[4]{2}\,e^{5i\pi/8} = \sqrt[4]{2}\,e^{-i\pi}e^{5i\pi/8} = \sqrt[4]{2}\,e^{-3i\pi/8}\,, \end{cases} \tag{4}$$

where I used that $-1 = e^{-i\pi} = e^{i\pi}$, but I chose the minus sign in the exponent to conform to WolframAlpha's single root.

My Mathematica program that calculates square roots of complex numbers, gave me two roots:

$$z^{1/2} = \begin{cases} \sqrt{\frac{1}{2}(-1+\sqrt{2})} - i\sqrt{\frac{1}{2}(1+\sqrt{2})}\,, \\ -\sqrt{\frac{1}{2}(-1+\sqrt{2})} + i\sqrt{\frac{1}{2}(1+\sqrt{2})}\,. \end{cases} \tag{5}$$

So, the conclusion is that the square roots of complex numbers can be quite complicated, even when the complex number looks 'simple'.