Math Diversion Problem 439

P. Reany

February 28, 2025

I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

The YouTube video is found at:

```
Source:https://www.youtube.com/watch?v=bU0-PYjyPPQ&t=1s
Title: A Really Cool Exponential Equation
 Presenter: SyberMath
```

1 The Problem

Given the relation

$$(\log x)^{\ln x} = x\,, \tag{1}$$

find the values of x.

2 The Solution

The natural action to begin with is to take the logarithm across (1):

$$(\ln x)\ln(\log x) = \ln x\,, \tag{2}$$

which has the trivial solution $x = 1$, but we have to reject it because it won't work in (1).[1] Therefore, we can divide through by $\ln x$, to get

$$\ln(\log x) = 1\,. \tag{3}$$

Now, we raise e to the power of this last equation to get

$$\log x = e^1 = e\,. \tag{4}$$

Finally, we raise 10 to the power of this last equation to get

$$x = 10^e\,. \tag{5}$$

[1] We can't have 0^0.