Math Diversion Problem 441

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> Our greatest weakness lies in giving up. The most certain way to succeed is always to try just one more time.
> —Thomas Edison

The problem is found at:

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Source: https://indico.cern.ch/event/726779/contributions
/2991244/attachments/1642552/2727515
/complex_numbers_exercises.pdf
Title: Complex numbers- Exercises with detailed solutions
 Presenter:  CERN
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1 The Problem

Given the relation

$$\mathrm{Re}(z(1+i)) + z\overline{z} = 0\,, \tag{1}$$

find the values of z.

2 The Solution

On setting $z = a + bi$ and $z\overline{z} = r^2 = a^2 + b^2$, then (1) becomes:

$$a - b = -a^2 - b^2\,, \tag{2}$$

which we can also write as

$$a^2 + a + b^2 - b = 0\,. \tag{3}$$

Now, if we can remember how to 'complete the squares', we can solve this for a convenient equation for its locus of points in ab-plane.

$$a^2 + a + \tfrac{1}{4} + b^2 - b + \tfrac{1}{4} = \tfrac{1}{2}\,, \tag{4}$$

or

$$(a + \tfrac{1}{2})^2 + (b - \tfrac{1}{2})^2 = \tfrac{1}{2}\,, \tag{5}$$

which describes a circle of radius $\sqrt{2}/2$, centered at point $(-\frac{1}{2}, \frac{1}{2})$.