Math Diversion Problem 456

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> Finally, brethren, whatsoever things are true, whatsoever things are honest, whatsoever things are just, whatsoever things are pure, whatsoever things are lovely, whatsoever things are of good report; if there be any virtue, and if there be any praise, think on these things.
> — A Bible Verse for a healthier mindset

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=iUK4Ik5I5oQ
Title: Harvard University logarithmic Problem.
 Presenter: Super Academy
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1 The Problem

Given the relation

$$x^{\log_5 3} = \sqrt{x} + 4x\,, \tag{1}$$

find the values of x.

2 The Solution

Let's try an α substitution to (1). Let

$$x = 5^{\alpha}\,, \tag{2}$$

then (1) becomes

$$3^{\alpha} = 5^{\alpha/2} + 4\,. \tag{3}$$

Let's try some small integer values for α, such as $\alpha = 2$:

$$3^2 = 5^{2/2} + 4\,, \tag{4}$$

which works; therefore,

$$x = 25\,. \tag{5}$$