Math Diversion Problem 459

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> It's clear we are at a place in scientific history
> where the interplay of the ideas between physics
> and number theory is quite important.
> — Minhyong Kim
> (Connecting number theory to physics,
> Quanta Magazine)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=5YAurLmKT4o
Title: JEE Mains Complex Logarithm Equation
 Presenter: maths & Olympiad
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1 The Problem

Given the relation

$$\log_3[7 + \log_4\{13 + \log_2(x+4)\}] = 2\,, \tag{1}$$

find the values of x.

2 The Solution

We should begin by raising 3 the power of Equation (1), yielding[1]

$$7 + \log_4\{13 + \log_2(x+4)\}] = 3^2 = 9\,, \tag{2}$$

or

$$\log_4\{13 + \log_2(x+4)\} = 2\,, \tag{3}$$

Next, we raise 4 to the power this last equation, to get

$$13 + \log_2(x+4) = 16\,, \tag{4}$$

[1] To raise a number b to the 'power of an equation' simply means this: If the equation is 'LHS = RHS', then $b^{\text{LHS=RHS}}$ means $b^{\text{LHS}} = b^{\text{RHS}}$.

or

$$\log_2(x+4) = 3\,, \tag{5}$$

Finally, we raise 2 to the power this last equation, to get

$$x + 4 = 8\,. \tag{6}$$

Hence,

$$x = 4\,. \tag{7}$$