Math Diversion Problem 466

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> You don't understand anything until you learn
> it more than one way.
> — Marvin Minsky

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=GluuMUWL1GY
Title: Harvard University Exponential Question!
 Presenter: Maths & Olympiad
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1 The Problem

Given the relation

$$9^{x-9} - 9^{y-9} = 6560\,, \tag{1}$$

find the integer values of x, y.

2 The Solution

My inclination is to multiply (1) through by 9^9:

$$9^x - 9^y = 6560 \times 9^9\,. \tag{2}$$

Now, since the RHS of this last equation is positive, then so it the LHS, which forces $x > y$, hence we can factor, to get

$$9^y(9^{x-y} - 1) = 6560 \times 9^9\,. \tag{3}$$

Obviously, we should at least try $y = 9$, then, after some cancellation,

$$9^{x-y} - 1 = 6560\,, \tag{4}$$

or

$$9^{x-y} = 6561 = 9^4\,. \tag{5}$$

With $y = 9$, then $x = 13$.