# Math Diversion Problem 492

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

The problem is found at:

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Source: https://indico.cern.ch/event/726779/contributions
/2991244/attachments/1642552/2727515
/complex_numbers_exercises.pdf
Title: Complex numbers- Exercises with detailed solutions
 Presenter:  CERN
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## 1 The Problem

Prove that there is no complex number such that

$$|z| - z = i\,. \tag{1}$$

Hint: $z$ will be represented as $z = a + bi$, where $a, b$ are real numbers. If we can't solve for $a, b$ as real numbers, that's a contradiction. Or, if we end up with something like $0 = 1$, that's also a contradiction. Either way, no solution.

## 2 The Solution

First, we know that $|z| = r$. Therefore, (1) takes the form

$$r - z = i\,. \tag{2a}$$

Let's add to this the complex conjugate, to get

$$r - \overline{z} = -i\,. \tag{2b}$$

On adding these together, we have that

$$2r - (z + \overline{z}) = 0\,, \tag{3}$$

which is better expressed as

$$2r - 2a = 0\,. \tag{4}$$

But this demands that we find a solution consistent with

$$r = a\,, \tag{5}$$

where

$$r^2 = a^2 + b^2\,. \tag{6}$$

Putting these together, we get that

$$b^2 = 0\,, \tag{7}$$

or simply

$$b = 0\,. \tag{8}$$

Using the results in (1) leads us to

$$a - a = i\,, \tag{9}$$

which is a contradiction.

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Alternatively, we can take the difference of (2a) and (2b), to get

$$-(z - \overline{z}) = 2i\,, \tag{10}$$

which simplifies to

$$-2ib = 2i\,, \tag{11}$$

implying that

$$b = -1\,. \tag{12}$$

From (6), we have that

$$r^2 = a^2 + 1\,. \tag{13}$$

But from (2a), we have that

$$r = a\,. \tag{14}$$

Combining these last two equations and simplifying, we get

$$a^2 = a^2 + 1\,, \tag{15}$$

or

$$0 = 1\,, \tag{16}$$

which is a contradiction.

So, we were not able to find solutions for $a$ and $b$.