

Math Diversion Problem 493

P. Reany

April 1, 2025

You don't understand anything until you learn
it more than one way.
— Marvin Minsky

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=sGLIBFkwnV4>
Title: A Cool Exponential Equation
Presenter: SyberMath

1 The Problem

Given the relation

$$6^x + 12^x = 24^x, \quad (1)$$

find the real values of x .

2 The Solution

My experience leads me to look for common factors to remove.

$$(2 \cdot 3)^x + (2 \cdot 2 \cdot 3)^x = (2 \cdot 2 \cdot 2 \cdot 3)^x, \quad (2)$$

So we can factor out $(2 \cdot 3)^x$, to get

$$1 + (2)^x = (2 \cdot 2)^x, \quad (3)$$

which we can reconfigure to

$$(2^x)^2 - (2^x) - 1 = 0. \quad (4)$$

So,

$$2^x = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}. \quad (5)$$

Hence,

$$x = \log_2 \left(\frac{1 + \sqrt{5}}{2} \right), \quad (6)$$

where we disregarded the negative sign to avoid a logarithm with a negative argument.