

Math Diversion Problem 502

P. Reany

April 6, 2025

To err is human; to really foul things up
requires partial differentiation.
— my adaptation of a quote from
Paul Ehrlich

1 The Problem

Source: <https://www.physicsforums.com/threads/for-this-partial-derivative-why-are-different-results-obtained.1053100/>

Title: For this Partial Derivative --
Why are different results obtained?

Presenter: Silvia2023

Given the relation

$$F(x, y) = Ax^2y, \quad (1)$$

find the derivative

$$\frac{dF}{d(1/x)}. \quad (2)$$

2 The Solution

We begin with the understanding that x and y are independent variables. Then, employing SD notation (but reverting to conventional notation wherever possible),

$$\frac{\delta F}{\delta(1/x)} = \frac{\partial F}{\partial x} \frac{\delta x}{\delta(1/x)} = (2Axy) \frac{dx}{d(1/x)} \quad (3)$$

$$= (2Axy) \left(\frac{d(1/x)}{dx} \right)^{-1} = (2Axy) (-x^{-2})^{-1} \quad (4)$$

$$= -2Ax^3y. \quad (5)$$