Math Diversion Problem 506

P. Reany

April 9, 2025

There is much you have to learn. You must explore; you
must reach out. Go...and give thought to the
the mysteries of the universe.
— The Galaxy Being
(An early proponent of
Life-Long Learning)

The problem is found at:

```
Source: https://www.youtube.com/watch?v=MCIyh1yVXNo
Title: 98% People Failed To Solve This!
 Presenter:  MathMinds
```

1 The Problem

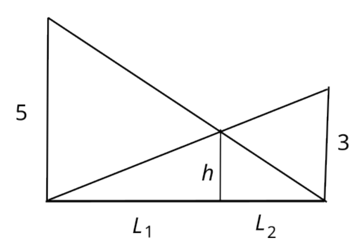

Figure 1. Depicted are two right triangles, one with side length 5, the other with side length 3. Both triangles share a common 'bottom' side of length $L_1 + L_2$. Use geometrical methods to solve for h, the length of the shortest vertical line segment.

2 The Preparation

This is all about similar triangles in a plane.

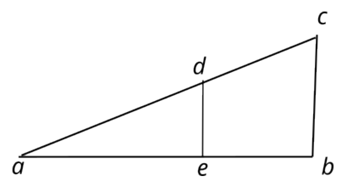

Figure 2. Depicted are two right triangles. We see that the smaller triangle can be rescale so as to fit perfectly over the larger triangle, because all the corresponding interior angles are congruent to each other in pairs.

Referencing Fig. 2, we see that triangles *cba* and *dea* are similar. Therefore, the ratios of corresponding parts are equal to a fixed scale factor. We don't care in this problem what that scale factor is; all we care about is that we can set up a proportionality, thusly,

$$\frac{cb}{ba} = \frac{de}{ea}\,. \tag{1}$$

And many other proportionalities can be inferred from the figure.

Note: Although I have demonstrated proportionality by comparing two right triangles, the triangles under comparison need not have any 'special' values for their interior angles.

3 The Solution

This is all about the use of similar triangles: Two triangles in a plane are similar if they differ only by a rotation and a rescaling of one of them.[1]

All we need is two proportions.

$$\frac{h}{L_1} = \frac{3}{L_1 + L_2}\,. \tag{2a}$$

$$\frac{h}{L_2} = \frac{5}{L_1 + L_2}\,, \tag{2b}$$

which can be rewritten as

$$\frac{h(L_1 + L_2)}{3} = L_1\,. \tag{3a}$$

$$\frac{h(L_1 + L_2)}{5} = L_2\,. \tag{3b}$$

Next, we just add these two equations together:

$$\frac{h(L_1 + L_2)}{3} + \frac{h(L_1 + L_2)}{5} = L_1 + L_2\,, \tag{4}$$

[1] Whether I should include reflections in this, I don't know.

On canceling the factor $L_1 + L_2$, we get

$$h \left[\frac{1}{3} + \frac{1}{5} \right] = 1 \, . \tag{5}$$

Solving for h, we have that

$$h = \frac{15}{8} \, . \tag{6}$$