Math Diversion Problem 508

P. Reany

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> He who would ride two camels, finds he can ride neither.
>
> — From an old movie

The problem is found at:

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Source: https://www.youtube.com/watch?v=DVbvcKsmqbs
Title: A factorial exercise
 Presenter:  Prime Newtons
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1 The Problem

Prove the relation

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}. \tag{1}$$

2 The Solution

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-(k-1))!} \tag{2}$$

$$= \frac{n!}{(k-1)!}\left[\frac{1}{k(n-k)!} + \frac{1}{(n+1-k)!}\right] \tag{3}$$

$$= \frac{n!}{(k-1)!}\left[\frac{(n+1-k)}{k(n-k)!(n+1-k)} + \frac{k}{k(n+1-k)!}\right] \tag{4}$$

$$= \frac{n!}{k!(n+1-k)!}\left[\frac{(n+1-k)}{1} + \frac{k}{1}\right] \tag{5}$$

$$= \frac{n!(n+1)}{k!(n+1-k)!} = \frac{(n+1)!}{k!(n+1-k)!} \tag{6}$$

$$= \binom{n+1}{k} \tag{7}$$