Math Diversion Problem 512

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A clue is anything that doesn't happen
the way it oughtta happen.
— Harry Orwell, the
TV show *Harry O*

The problem is found at:

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Source: https://www.youtube.com/watch?v=HXVlIEnbeLg
Title: How to solve for "x"
 Presenter:  Math Beast
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1 The Problem

Given the relation

$$e^x = x^2\,, \tag{1}$$

find the values of x.

2 The Preparation

I intend to use the Lambert W function Lemma, that, for $a > 0$, given

$$za^z = B\,, \tag{2}$$

then

$$z = W_a(B)\,, \tag{3}$$

where

$$W_a(B) \equiv \frac{W(B\ln a)}{\ln a}\,, \tag{4}$$

which becomes the ordinary Lambert W function when $a = e$.

3 The Solution

Let's begin by taking the square root across the Given:

$$(e^{1/2})^x = \pm x\,. \tag{5}$$

Then a little algebraic manipulation.

$$1 = \pm x(e^{1/2})^{-x}\,. \tag{6}$$

Flipping sides,

$$-x(e^{1/2})^{-x} = \mp 1\,. \tag{7}$$

On applying the lemma from the previous section, we have that

$$-x = W_{e^{1/2}}(\mp 1) = \frac{W_n(\pm 1 \cdot \ln e^{1/2})}{\ln e^{1/2}} = \frac{W_n(\pm\frac{1}{2})}{1/2}\,. \tag{8}$$

Or,

$$x = -2W_n(\pm\tfrac{1}{2})\,, \tag{9}$$

with real solution

$$x = -2W_0(\tfrac{1}{2}) = -0.703467\,, \tag{10}$$

which was provided by WolframAlpha.