Math Diversion Problem 513

P. Reany

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It is clear that the chief end of mathematical study
must be to make the students think.
— John Wesley Young
(So think!)

The problem is found at:

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Source: https://cs.uwaterloo.ca/research/tr/1993/03/W.pdf
Title:  'On the Lambert W Function'
 Presenter:  Corless, Gonnet, Hare, Jeffrey, Knuth
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From: *Advances in Computational Mathematics*, Vol 5 (1996) 329–359, R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffrey, and D.E. Knuth.

1 The Problem

Given the relation

$$c = q(1 + e^{-cR}) \,, \tag{1}$$

find the values of c.[1]

2 The Solution

My first action is to introduce a variable change. Let

$$\lambda = cR \,. \tag{2}$$

The (1) becomes

$$\frac{\lambda}{qR} - 1 = e^{-\lambda} \,. \tag{3}$$

Now let

$$\omega = \frac{\lambda}{qR} - 1 \,, \tag{4}$$

[1] This equation is found on page 335. Solving for the '+' should be enough.

then (3) becomes

$$\omega = e^{(\omega+1)qR}\,, \tag{5}$$

which can be manipulated to

$$\omega e^{qR\omega} = e^{-qR}\,. \tag{6}$$

Multiplying through by qR, gives

$$qR\omega e^{qR\omega} = qRe^{-qR}\,. \tag{7}$$

We are now ready to apply the Lambert W function to both sides, to get

$$qR\omega = W(qRe^{-qR})\,, \tag{8}$$

or

$$\omega = \frac{1}{qR}W(qRe^{-qR})\,. \tag{9}$$

Now we back substitute.

$$\frac{\lambda}{qR} - 1 = \frac{1}{qR}W(qRe^{-qR})\,. \tag{10}$$

Hence, from (2),

$$\frac{c}{q} = 1 + \frac{1}{qR}W(qRe^{-qR})\,. \tag{11}$$

And finally,

$$c = q + \frac{1}{R}W(qRe^{-qR})\,. \tag{12}$$