Math Diversion 524

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If you're not in love with the Truth, you could be
talked into believing almost anything.
— Author

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=UWD7Z5iMyYk
Title:A Nice Algebra Problem | Math Olympiad |
 Presenter: SALogic
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1 The Problem

Given the relation

$$\left(\frac{\sqrt{x}}{x}\right)^{x-1} = \left(\frac{x}{\sqrt{x}}\right)^{x-3}, \tag{1}$$

find the positive real values of x.

2 The Solution

We can simplify (1) down to,

$$\left(x^{-\frac{1}{2}}\right)^{x-1} = \left(x^{\frac{1}{2}}\right)^{x-3}. \tag{2}$$

Before we compare exponents, is there a trivial solution? There is! Namely,

$$x = 1\,. \tag{3}$$

On comparing exponents, we get

$$-\tfrac{1}{2}(x-1) = \tfrac{1}{2}(x-3)\,. \tag{4}$$

Solving for x, we get

$$x = 2\,. \tag{5}$$