Math Diversion Problem 527

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> He that answereth a matter before he heareth it,
> it is folly and shame unto him.
> — Proverbs 18:13

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Y8ob91CrCQc
Title:  I got this asked as a Harvard interview question
 Presenter: Higher Mathematics
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1 The Problem

Given the relation

$$\phi = \sqrt{3} - 1\,, \tag{1}$$

find the value of ϕ^{10}.

Note: The Presenter's version used the 8th power; I'm using the 10th power.

2 The Solution

My plan is first to calculate ϕ^5, simplify that, the find $\phi^{10} = (\phi^5)^2$. So, to take ϕ to the fifth power, I will need Pascal's coefficients:

$$1 \qquad 5 \qquad 10 \qquad 10 \qquad 5 \qquad 1 \tag{2}$$

Then,

$$\phi^5 = (\sqrt{3}\,)^5 - 5(\sqrt{3}\,)^4 + 10(\sqrt{3}\,)^3 - 10(\sqrt{3}\,)^2 + 5\sqrt{3} - 1 = -76 + 44\sqrt{3}\,. \tag{3}$$

Hence,

$$\phi^{10} = (-76 + 44\sqrt{3}\,)^2 = 11,584 - 6688\sqrt{3}\,. \tag{4}$$