Math Diversion Problem 533

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> Our greatest weakness lies in giving up. The
> most certain way to succeed is always to
> try just one more time.
> —Thomas Edison

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=_avyzbyAD9E
Title: A Nice Algebra Problem
 Presenter: SALogic
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1 The Problem

Given the relation

$$x^{1/3} + x^{1/2} = 12\,, \tag{1}$$

find the real values of x.

2 The Solution

Let's begin with the variable substitution: $x \longrightarrow y^6$. Then (1) becomes

$$y^2 + y^3 = 12\,. \tag{2}$$

y	$y^2 + y^3 - 12$
1	-10
2	0 ✓

Table 1: Heuristic: Solved by Table: Look for a zero entry.

$$y = 2 \quad \Longrightarrow \quad x = 2^6 = 64\,. \tag{3}$$

On using long division to divide $y^3 + y^2 - 12$ by $y - 2$, we get $y^2 + 3y + 6$, which has no real roots, and those roots won't generate additional real roots for $x = y^6$, either.