

Math Diversion Problem 537

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April 27, 2025

Keep an open mind. That's the secret.
— Doctor Who

The problem is found at:

Source: A common problem in group theory

1 The Problem

Let G be a group, and let f be a mapping of G to G , such that

$$f(g) = g^{-1}. \tag{1}$$

Show that f is a one-to-one map.

2 Solution

A mapping f is said to be one-to-one if, for all $g, h \in G$, whenever

$$f(g) = f(h), \tag{2}$$

then

$$g = h. \tag{3}$$

So, if we combine (2) and the definition of f as given, then

$$g^{-1} = h^{-1}. \tag{4}$$

Next, we multiply through on the left by g and on the right by h , to get

$$gg^{-1}h = gh^{-1}h. \tag{5}$$

On enforcing the group cancellation law, we have that

$$h = g, \tag{6}$$

which implies $g = h$, as we were to show.