Math Diversion Problem 544

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> A new scientific truth does not triumph by convincing its
> opponents and making them see the light, but rather
> because its opponents die and a new generation
> grows up that is familiar with it.
> — Max Planck

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=aTaEBktzFsE
Title: A Nice Algebra Problem | Math Olympiad
 Presenter: SALogic
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1 The Problem

Given the relation

$$9^{x+1} - 9^{x-1} = 20\,, \tag{1}$$

find the real values of x.

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2 The Solution

Let's begin with some factoring:

$$9^x(9 - 9^{-1}) = 20\,. \tag{2}$$

Next, we add a touch of cross multiplication with simplification:

$$9^x = \frac{20}{9 - 9^{-1}} = \frac{20}{\frac{80}{9}} = \frac{9}{4}\,. \tag{3}$$

Now, we can take a smidgen of a square root:

$$3^x = \frac{3}{2}\,, \tag{4}$$

which gives us the real solution by logarithms:

$$x = 1 - \frac{\log 2}{\log 3}\,. \tag{5}$$