Math Diversion Problem 545

P. Reany

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> You will never plough a field if you only
> turn it over in your mind.
> — Irish Proverb

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=7VOwC35DFZw
Title: Math Olympiad | A Nice Algebra Problem
 Presenter: MathMinds
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1 The Problem

Given the relations

$$2^a \cdot 5^b = 50\,, \tag{1a}$$

$$2^b \cdot 5^a = 20\,, \tag{1b}$$

find the integer values of a, b.

2 The Solution

If we multiply (1a) by (1b), and then if we divide them in that order, we get the pair of equations:

$$2^{a+b} \cdot 5^{a+b} = 1000\,, \tag{2a}$$

$$2^{a-b} \cdot 5^{-(a-b)} = 5/2\,. \tag{2b}$$

From (2a) we get

$$10^{a+b} = 10^3\,, \tag{3}$$

from which we get:

$$a + b = 3\,. \tag{4}$$

From (2b) we get

$$2^{a-b+1} = 5^{a-b+1}\,, \tag{5}$$

from which we get:

$$a - b + 1 = 0\,, \tag{6}$$

which is the only integer value that could work. The solution to (4) and (6) is

$$a = 1, \quad \text{and} \quad b = 2\,. \tag{7}$$