Math Diversion 547

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> There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.
> — Lord Kelvin

Problem found at:

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Source: University Physics (Sears & Zemansky, Addison Wesley 4th ed).
Title: A Neat Physics Problem (resistance network)
 Presenter: Patrick
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1 The Problem

This problem is from *University Physics* (Sears & Zemansky, Addison Wesley 4th ed), page 421: 29-10. It asks the student to prove that the resistance of the following network is equal to $(1+\sqrt{3})r$.[1]

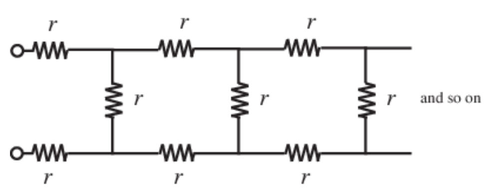

Figure 1

It took me a while to figure out that the best way to solve this problem is with self-similarity. But first we need some basics.

[1] This paper is a redo of an article that first appeared in the *Arizona Journal of Natural Philosophy*, April, 1991. The solution I present here is an improvement over my original solution.

2 The Preparation

This problem concern a resistor network in which resistors are connected to each other in both series and parallel fashion. The resistance of two resistors connected in series is just the sum of their individual resistances. It's a bit more complicated when they are connected in parallel, as is demonstrated in Fig. 0 below.

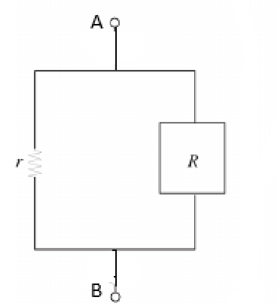

Figure 0. The two resistors r and R are said to 'in parallel'. See the formula below for their combined resistance.

The combined resistance of the two resistors in Fig. 0 is

$$R_{\text{combined}} = \frac{rR}{r+R} \,. \tag{1}$$

3 The Solution

The key to making an elegant solution to this problem is to notice that the subnetwork depicted in Fig. 2—that part being to the right of line segment AB—is equivalent to the entire network. Hence, self-similarity.

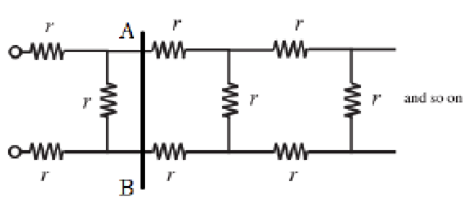

Figure 2. The line segment divides the full network into two subnetworks.

Frankly, I don't know if this network has a resistance or not. On the assumption that it does, let's call it R. Now, it's perfectly allowable to redraw the network, so long as we maintain the same network topology (i.e., maintaining what's connected to what). I offer the equivalent redrawn figure below.

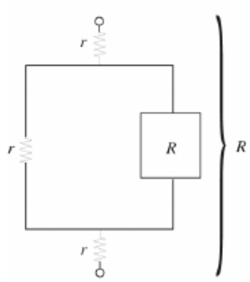

Figure 3. The redrawn network is now just a resistor r in series with R_{combined} in series with another r.

By our own resistance assignment, the total resistance of this mini-network is just R, but it also has to be equal to the sum of its parts, namely,

$$R = r + \frac{rR}{r+R} + r\,. \tag{2}$$

This equation can be solved for a quadratic in R:

$$R^2 - 2rR - 2r^2 = 0\,, \tag{3}$$

with real and positive solution for R:

$$R = (1+\sqrt{3})r\,. \tag{4}$$