Math Diversion 550

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> Wisdom is that ability to make decisions
> you won't regret in the long run.
> — The Author

Nature of the Problem: Continued fractions and self-similarity.

1 The Problem

What's the value of x defined in the continued fraction

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}} \quad ? \tag{1}$$

Here's where self-similarity comes in. Notice that the expression in the first-level denominator is itself equal to x. Thus, this last equation becomes

$$x = 1 + \frac{1}{x}\,, \tag{2}$$

which has solution[1]

$$x = \frac{1+\sqrt{5}}{2}\,, \tag{3}$$

which, by the way, is the Golden Ratio!

The take-away point here is that we should see an immediate analogy between the structure of the continued fraction in Eq. (1) and in the circuit diagram in Figure 1. I now assume that the authors of the problem had desired that the student would see the self-similarity aspect of the circuit and imagine

[1] We have to ignore the negative root, of course.

the subnetwork as shown below in Figure 1, as the portion of the full network to the right of line segment AB, is self-similar to the entire network:

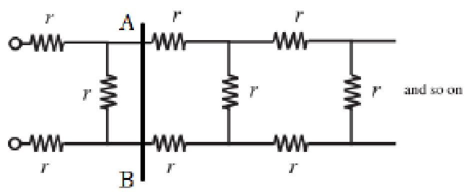

Figure 1. The line segment divides the full network into two subnetworks. This network problem was worked out a few problems back .

Two points left to make: First, it's not a good idea to become too narrowly focused in one's study of mathematics. Second, one may find, in the process of time, that a field of mathematics that seems quite unrelated to some other field of study, may turn out to have a profound connection to it.