

# Math Diversion Problem 553

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May 3, 2025

People often overlook the obvious.  
— Doctor Who

The problem is found at:

Source: <https://www.youtube.com/watch?v=GJcHkq7HzgM>

Title: SAT Geometry Problem

Presenter: Brain Station

## 1 The Problem

Determine the value of  $x$  from the given information in the figure.

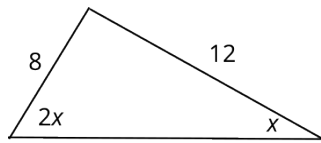


Figure 1. How would you solve for  $x$ ? I hope not by the Law of Cosines.

## 2 The Preparation

The Law of Sines concerning triangles:

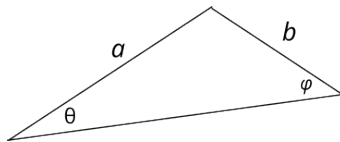


Figure 2. The Law of Sines requires two interior angles and their opposite sides.

The Law of Sines is the following rule that relates two interior angles of a triangle (which two doesn't matter) and their opposite sides, as follows:

$$\frac{\sin \varphi}{a} = \frac{\sin \theta}{b}, \quad (1a)$$

or

$$\frac{\sin \varphi}{\sin \theta} = \frac{a}{b}. \quad (1b)$$

We also will need a double-angle identity for sine:

$$\sin 2x = 2 \sin x \cos x. \quad (2)$$

### 3 The Solution

We apply the Law of Sines to triangle to get

$$\frac{\sin 2x}{\sin x} = \frac{12}{8} = \frac{3}{2}. \quad (3)$$

But drawing on (2), we have that

$$2 \cos x = \frac{3}{2}. \quad (4)$$

Hence,

$$x = \cos^{-1} \frac{3}{4}. \quad (5)$$