Math Diversion 557

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Theory like mist on eyeglasses — obscures facts.
— Charlie Chan

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=KhT6IT2viDg
Title: Final Result Will Blow Your Mind!
 Presenter: Brain Station Videos
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1 The Problem

Given the relation

$$\phi = \sqrt[3]{7+\sqrt{50}} + \sqrt[3]{7-\sqrt{50}}\,, \tag{1}$$

find the real values of ϕ.

2 The Preparation

Line 4 of Pascal's triangle is: 1 3 3 1.

3 The Solution

Let's begin by cubing ϕ:

$$\begin{aligned}\phi^3 &= \left(\sqrt[3]{7+\sqrt{50}}\,\right)^3 + 3\left(\sqrt[3]{7+\sqrt{50}}\right)^2\left(\sqrt[3]{7-\sqrt{50}}\right)^1 && \text{(2a)}\\ &\quad + 3\left(\sqrt[3]{7+\sqrt{50}}\right)^1(\sqrt[3]{7-\sqrt{50}}\Big)^2 + \left(\sqrt[3]{7-\sqrt{50}}\right)^3 && \text{(2b)}\\ &= 14 - 3\phi\,, && \text{(2c)}\end{aligned}$$

and I'm leaving it to the interested reader to fill in the missing steps. But this gives us the cubic equation:

$$\phi^3 + 3\phi - 14 = 0\,, \tag{3}$$

which has the real solution $\phi = 2$, which can be discovered by inspection. But does (3) have any other real solutions? Using long division, I got the factorization

$$(\phi - 2)(\phi^2 + 2\phi + 7) = 0\,. \tag{4}$$

However, the roots to

$$\phi^2 + 2\phi + 7 = 0\,, \tag{5}$$

are nonreal, so we are done.