Math Diversion 564

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They that observe lying vanities forsake
their own mercy.
— Jonah 2:8

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Source: https://www.youtube.com/watch?v=HM1KagtoRuk
Title: A Harder Math Question from China Elementary School
 Presenter: Math Beast
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1 The Problem

Given the relation

$$1+\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}}=\frac{17}{10}\,, \tag{1}$$

where a, b, c are integers, solve for a, b, c.

2 The Solution

Maybe the first rule of problem-solving should be to do the obvious stuff first. In this case, (1) becomes

$$\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}}=\frac{7}{10}\,. \tag{2}$$

But wait! We have one more obvious step: Inversion!

$$a+\cfrac{1}{b+\cfrac{1}{c}}=\frac{10}{7}\,. \tag{3}$$

At some point we have to use the fact that a, b, c are integers to our advantage, so let's do that now. Apply a bit of arithmetic to get:

$$\frac{7}{b + \frac{1}{c}} = 10 - 7a\,. \tag{4}$$

Now, since the RHS is an integer, the LHS must also be an integer, therefore,

$$b + \frac{1}{c} = \frac{7}{d}\,, \tag{5}$$

where d is some integer. Once we find d, everything else should fall out easily. Let's re-express (4) in terms of a and d:

$$d = 10 - 7a\,. \tag{6}$$

By inspection, we can get a solution for a by choosing $d = 3$. Let's go ahead and do that and see where that takes us.

$$a = 1\,. \tag{7}$$

On using $d = 3$ in (5), we get

$$b + \frac{1}{c} = \frac{7}{3}\,, \tag{8}$$

With a little arithmetic, we get

$$7 - 3b = \frac{3}{c}. \tag{9}$$

Again, requiring that both sides be integers, gives us

$$c = 3\,. \tag{10}$$

But then (9) gives us

$$b = 2\,. \tag{11}$$

Hence, the full solution is

$$a = 1\,, \qquad b = 2\,, \qquad c = 3\,. \tag{12}$$