Math Diversion Problem 571

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> Learning is a treasure that will follow its owner everywhere.
> — Chinese proverb

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=bLKECNiNiF8
Title: Solving log values with different bases
 Presenter:  Math Beast
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1 The Problem

Given the relation

$$\log_4 x + \log_{16} x + \log_2 x = 7\,, \tag{1}$$

find the real values of x.

2 The Preparation

Lemma: Fundamental Rule of Logarithmic Conversion (base a to base 10):

$$\log_a x = \frac{\log x}{\log a}\,. \tag{2}$$

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3 The Solution

The Given can be rewritten as

$$\log x \left(\frac{1}{\log 4} + \frac{1}{\log 16} + \frac{1}{\log 2}\right) = 7\,, \tag{3}$$

But

$$\begin{aligned}\frac{1}{\log 4}+\frac{1}{\log 16}+\frac{1}{\log 2} &= \frac{1}{2\log 2}+\frac{1}{4\log 2}+\frac{1}{\log 2}\\ &= \frac{1}{\log 2}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{1}\right)\\ &= \frac{1}{\log 2}\left(\frac{7}{4}\right), \end{aligned} \tag{4}$$

Thus (3) becomes

$$\log x = 4\log 2 = \log 2^4 = \log 16\,. \tag{5}$$

On raising 10 to this last equation, we get

$$x = 16\,. \tag{6}$$