Math Diversion Problem 579

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May 11, 2025

No mystery is closed to an open mind.
— Sightings TV show

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=O8rdQyUzhgU
Title: Harvard University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relation

$$6 = 216^{8^{3x^{-1}}}\,, \tag{1}$$

find all real values of x.

2 The Solution

(Note: $216 = 6^3$.)

Let's rewrite (1) into the form:

$$6 = 6^{3\cdot 8^{3x^{-1}}}\,, \tag{2}$$

On equating exponents, we get

$$3 \cdot 8^{3x^{-1}} = 1\,, \tag{3}$$

or

$$8^{3x^{-1}} = \frac{1}{3}\,. \tag{4}$$

Then we take the natural logarithm, to get

$$3x^{-1} \ln 8 = \ln 1/3 = -\ln 3\,. \tag{5}$$

Solving for x, we get

$$x = -\frac{3\ln 8}{\ln 3} = -\frac{9\ln 2}{\ln 3}\,. \tag{6}$$