Math Diversion Problem 580

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May 11, 2025

Theory like mist on eyeglasses — obscures facts.
— Charlie Chan

1 The Problem

In what ratio should water be added to a liquid costing \$12 per liter so as to make a profit of 25% by selling the diluted liquid at \$13.75 per liter?

We'll worry about the ratio after we have calculated how much water should be added to the starting liquid, which we'll set at 1 liter. We lose no generality by doing this.

In the graphic below, we show a 'before and after' process of adding water to this starting liquid. We will neglect the price of the water as a first approximation.

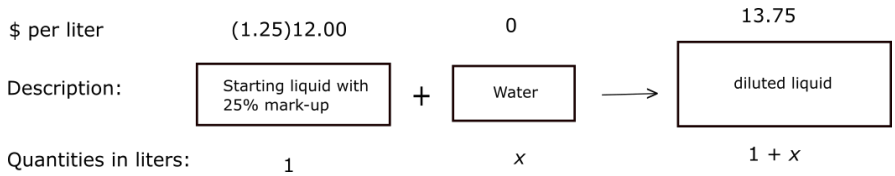

Figure 1. This graphic represents the adding some water x to a starting liquid in a 'before and after' process.

We begin with the cost conservation equation:

$$(1.25)(\$12.00/\text{liter})(1\,\text{liter}) + (\$0.00/\text{liter})(x\,\text{liter}) = (\$13.75/\text{liter})(1+x)\text{liter}\,. \tag{1}$$

Solving this, $x = 0.090909....$ But we are asked to find the ratio of $x : 1$, which is $0.090909 : 1$, or (approximately) $1 : 11$.