Math Diversion Problem 581

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Every big idea needs someone to defend it.
— Cybersecurity

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=JmyfgUglUzA
Title: Unlocking Trigonometric Secrets
 Presenter: Numbers & Numbers
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1 The Problem

Given the relation

$$\phi_+ = \sin\theta + \cos\theta = \frac{7}{5}\,, \tag{1}$$

find all real values of

$$\phi_- = \sin\theta - \cos\theta\,. \tag{2}$$

2 The Solution

Note:

$$\phi_+^2 = (\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta\,, \tag{3}$$

$$\phi_-^2 = (\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta\,. \tag{4}$$

Now,

$$\phi_+^2 + \phi_-^2 = 2\,. \tag{5}$$

Therefore,

$$\phi_-^2 = 2 - \phi_+^2 = 2 - \frac{49}{25} = \frac{1}{25}\,. \tag{6}$$

Hence,

$$\phi_- = \pm\frac{1}{5}\,. \tag{7}$$