Math Diversion Problem 585

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Hasty decisions are the makings for eternal regrets.
— The Author

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=OF_MA2bRTMI
Title: Can You Solve This?
 Presenter:  Brain Station Advanced
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1 The Problem

In the graphic below, we are given enough information to solve for x, which is the side length of a square. What is the area of the square?

The area of the square is, of course, just x^2, but how do we arrive at that?

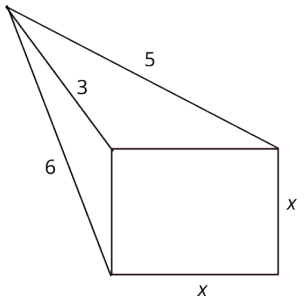

Figure 1. Should we use Heron's Formula, the cross product, or something else?

Actually, let's try something else: Once we've coordinatized the vertices, we can use the distance formula. So, what does that look like. See Fig. 2!

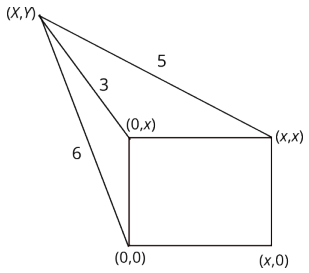

Figure 2. Do you see how to proceed from here?

If we use the distance formula for the line segments labeled $6, 3, 5$, we will obtain three equations in the three unknowns x, X, Y. So, let's do it:

$$X^2 + Y^2 = 6^2\,, \tag{1a}$$

$$X^2 + (Y - x)^2 = 3^2\,, \tag{1b}$$

$$(X - x)^2 + (Y - x)^2 = 5^2\,, \tag{1c}$$

with constraints: $x > 0$, $X < 0$, and $Y > 0$.

If you want to solve these by hand, go right ahead, but I used WolframAlpha, to get

$$x \approx 3.0938 \qquad \text{and} \qquad x^2 \approx 9.572\,. \tag{2}$$