

Math Diversion Problem 588

P. Reany

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Don't ever take a fence down until you know
the reason it was put up.
— Chesterton

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=pMGz4fVXqGE>
Title: Are YOU smart enough to get into Cambridge?
Presenter: Math Queen

1 The Problem

Given the relation

$$3^x - (\sqrt{3})^{x+4} + 20 = 0, \quad (1)$$

find the sum of the real solutions to x .

2 The Solution

Some short steps:

$$3^x - 9(\sqrt{3})^x + 20 = 0, \quad (2)$$

$$3^x - 9(3^x)^{1/2} + 20 = 0. \quad (3)$$

Solving for $(3^x)^{1/2}$ roots with the quadratic formula:

$$(3^x)^{1/2} = 5, 4. \quad (4)$$

On squaring, we have that

$$3^x = 25, 16. \quad (5)$$

Take these to the logarithm base 3:

$$x = \log_3 25, \log_3 16. \quad (6)$$

And finally, the Sum of the solutions:

$$\text{Sum} = \log_3 25 + \log_3 16 = \log_3 (25 \cdot 16) = \log_3 (5 \cdot 4)^2 = 2 \log_3 20. \quad (7)$$