

Math Diversion Problem 589

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First things first...But not necessarily in that order.

— Doctor Who

The YouTube video is found at:

Source: https://www.youtube.com/watch?v=LK-9oM_BQh4

Title: Can You Solve This Math Olympiad?

Presenter: Brain Station Advanced

1 The Problem

Given the relation

$$2^a + 2^b + 2^c = 148, \quad (1)$$

find the positive integer values of a, b, c , subject to the constraint: $a > b > c$.

2 The Solution

Since all the exponents are greater than or equal to unity, then each of the three terms is a power of 2. Thus, the building blocks for this summation are powers of 2, as listed below:

$$2, 4, 8, 16, 32, 64, 128, \quad (2)$$

and that is as high as we can go and remain less than 148.

So, we need to find three of these powers of 2 that sum to 148. Let's start with one of them being 128 and see if we can find two more that add to twenty. In fact, we can!

$$128 + 16 + 4 = 148, \quad (3)$$

or

$$2^7 + 2^4 + 2^2 = 148, \quad (4)$$

giving us the solution:

$$a = 7, \quad b = 4, \quad c = 2. \quad (5)$$

If you continue with this procedure, you will find that there are no other solutions.