Math Diversion Problem 593

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> I think that mathematics can benefit by acknowledging that the creation of good models is just as important as proving deep theorems.
> — David Mumford

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=OTGiyXfCu5Q
Title: Can you solve this Integration Problem
 Presenter: Ankit Physics Gurukul
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1 The Problem

Integrate the following integral:

$$I = \int \frac{x}{3-2x}\,dx\,. \tag{1}$$

2 The Solution

The first step is to modify the numerator:

$$\begin{aligned} I &= -\tfrac{1}{2}\int \frac{2x}{2x-3}\,dx && \text{(2a)}\\ &= -\tfrac{1}{2}\left[\int \frac{2x-3}{2x-3}\,dx + \int \frac{3}{2x-3}\,dx\right] && \text{(2b)}\\ &= -\tfrac{1}{2}\left[\int dx + \tfrac{3}{2}\int \frac{d(2x-3)}{2x-3}\right] && \text{(2c)}\\ &= -\tfrac{1}{2}\left[\,x + \tfrac{3}{2}\ln\left|2x-3\right|\,\right] + C\,. && \text{(2d)} \end{aligned}$$