Math Diversion Problem 594

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> When I give this talk to a physics audience, I
> remove the quotes from my 'Theorem'.
> — Brian Greene

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=tmjI6P96bVU
Title: Can you solve this Integration Problem
 Presenter: Ankit Physics Gurukul
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1 The Problem

Given the relation:

$$t^{\sqrt{t}} = \sqrt{t^t}\,, \tag{1}$$

solve for the values of t.

2 The Solution

The first step is to get rid of the square root on the RHS:

$$t^{2\sqrt{t}} = t^t\,. \tag{2}$$

Next, we take the natural logarithm across this equation.

$$2\sqrt{t}\ln t = t\ln t\,. \tag{3}$$

Clearly, $\mathbf{t = 1}$ satisfies this equation but also (1), so it's a solution. Are there any more?

On dividing through by $\ln t$ and simplifying, we get

$$t^2 - 4t = 0\,, \tag{4}$$

with obvious solutions $t = 0$ and $\mathbf{t = 4}$. That latter solution also works in (1), but the former does not.