# Math Diversion Problem 595

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> Mathematics compares the most diverse phenomena and
> discovers the secret analogies that unite them.
> — Joseph Fourier

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=MqVACSgL1HQ
Title: Austrailia Can you solve this partial fraction integral
 Presenter: Ankit Physics Gurukul
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## 1 The Problem

Integrate the following integral:

$$I = \int \frac{3x^2}{1+3x^2}\,dx\,. \tag{1}$$

## 2 The Solution

The first step is to virtually emplace a '1' in the numerator:

$$\begin{aligned} I &= \int \frac{3x^2}{1+3x^2}\,dx && \text{(2a)}\\ &= \int \frac{1+3x^2}{1+3x^2}\,dx - \int \frac{1}{1+3x^2}\,dx && \text{(2b)}\\ &= \int dx - \int \frac{1}{1+(\sqrt{3}x)^2}\,dx && \text{(2c)}\\ &= x - \frac{1}{\sqrt{3}}\int \frac{1}{1+(\sqrt{3}x)^2}\,d(\sqrt{3}x) && \text{(2d)}\\ &= x - \frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x) + C\,. && \text{(2e)} \end{aligned}$$