# Math Diversion 598

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A clue is anything that doesn't happen
the way it oughtta happen.
— Harry Orwell, TV
show *Harry O*

**Nature of the Problem:** Identity in logarithms.

## 1 The Problem

Prove the identity:

$$\log_{n^k} a^k = \log_n a \tag{1}$$

## 2 The Preparation

A few useful lemma:

$$\log_n x^w = w \log_n x\,. \tag{2}$$

$$\log_\ell B = \frac{\log_m B}{\log_m \ell}\,. \tag{3}$$

$$\log_n n = 1\,. \tag{4}$$

## 3 The Solution

$$\begin{aligned}
\log_{n^k} a^k &= k \log_{n^k} a \\
&= k \frac{\log_n a}{\log_n n^k} \\
&= k \frac{\log_n a}{k \log_n n} \\
&= \frac{\log_n a}{\log_n n} \\
&= \log_n a
\end{aligned} \tag{5}$$