Math Diversion Problem 599

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> Mathematics is the art of reducing any
> problem to linear algebra.
> — William Stein

The YouTube video is found at:

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Source: https://www.youtube.com/shorts/swZmfHgE_UU
Title: The trickiest problem on the SAT
 Presenter: YourSATCoach (shorts)
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1 The Problem

Given the relation

$$x^x = 2^{2048}\,, \tag{1}$$

find the real values of x.

Hint: If you're going to solve this kind of problem on a timed exam, you should memorize all powers of 2 up to $2^{11} = 2048$.

2 The Solution

I'll start off with a standard α transformation:

$$x = 2^\alpha\,. \tag{2}$$

Then (1) becomes

$$2^{\alpha 2^\alpha} = 2^{2^{11}}\,. \tag{3}$$

On equating exponents, we get

$$\alpha 2^\alpha = 2^{11}\,. \tag{4}$$

We could solve this using the Lambert W function, but I'll use a table this time.

α	$\alpha 2^\alpha$
2^2	$2^2 \cdot 2^4 = 2^6$
2^3	$2^3 \cdot 2^8 = 2^{11}$ ✓

Table 1: Alpha has to be a power of two!

Thus,

$$\alpha = 2^8 \,, \tag{5}$$

so

$$x = 2^8 = 256 \,. \tag{6}$$