

Math Diversion Problem 600

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To guard against a problem [occurring], you need
first to imagine that it can happen.
— Dave Farley

The YouTube video is found at:

Source: https://www.youtube.com/shorts/swZmfHgE_UU
Title: The trickiest problem on the SAT
Presenter: YourSATCoach (shorts)

1 The Problem

Given the relation

$$x^x = 2^{2048}, \quad (1)$$

find the real values of x .

2 The Solution

This is the same problem as the last one, but this time I will solve it using the Lambert W function instead of an α transformation. See my short write-up paper by a link on it for help on the Lambert W function.

I'll start off with the substitution: $2^{2048} = 2^{2^{11}}$. Then (1) becomes

$$x^x = 2^{2048} = 2^{2^{11}}. \quad (2)$$

On taking the natural logarithm, we get

$$x \ln x = 2^{11} \ln 2 = 2^8 \ln 2^3 = 2^8 \ln 2^8. \quad (3)$$

On taking the Lambert W function on this, we have that

$$\ln x = \ln 2^8. \quad (4)$$

On raising e to the power of this last equation, we get

$$x = 2^8 = 256. \quad (5)$$