Math Diversion Problem 604

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> Zathras is used to being beast of burden to other
> people's needs. Very sad life... Probably have
> very sad death. But, at least
> there is symmetry.
> —Zathras
> A character from *Babylon 5*

The YouTube video is found at:

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Source: The Ether of Mathematical Ideas
 Presenter: Patrick
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1 The Problem

Show that

$$\int \frac{e^x+1}{e^x-1}\,dx = 2\ln\left|\sinh\frac{x}{2}\right| + C\,. \tag{1}$$

2 The Solution

The first step is to multiply numerator and denominator by $e^{-x/2}$:

$$\int \frac{e^x+1}{e^x-1}\,dx = \int \frac{e^{x/2}+e^{-x/2}}{e^{x/2}-e^{-x/2}}\,dx \tag{2a}$$

$$= \int \frac{\cosh x/2}{\sinh x/2}\,dx \tag{2b}$$

$$= 2\int \frac{\cosh x/2}{\sinh x/2}\,d(x/2) \tag{2c}$$

$$= 2\int D_{x/2}[\ln(\sinh x/2)]\,d(x/2) \tag{2d}$$

$$= 2\ln\left|\sinh\frac{x}{2}\right| + C\,. \tag{2e}$$